NEUTRINO SCATTERING IN A MAGNETIC FIELD

K.S. McFarland[1], A.C. Melissinos[1]
N.V. Mikheev[2] and W.W. Repko[3]

[1]Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA
[2]Department of Physics, Yaroslavl' State University, Yaroslavl', 150000 Russia
[3]Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA

July 6, 2001

Abstract

Motivated by the evidence for a finite neutrino mass we examine anew the interaction of neutrinos in a magnetic field. We present the rate for radiative scattering for both massless and massive neutrinos in the standard model and give the corresponding numerical estimates. We also consider the effects arising from a possible neutrino magnetic moment.

1. Introduction

Neutrino flavor oscillations, for which there now appears significant experimental evidence, have established that at least some neutrino has non-zero mass. However the sensitivity to mass in vacuum flavor oscillations only enters as Δm^2, and thus one is unable to establish the neutrino's mass. Electromagnetic interactions of neutrinos, however, often have direct sensitivity to the neutrino mass. In this paper we evaluate a number of electromagnetic processes that could result when a high energy neutrino enters a laboratory magnetic field and consider the practicality of observing such effects.

We first consider the radiative scattering of high energy neutrinos in a magnetic field. Such scattering arises from the coupling to the field of the virtual charged particles in the loop diagrams shown in Fig.1. Radiative scattering can also be viewed as the back-scattering of the virtual photons in the magnetic field, off the high energy neutrinos.

Neutrino photon scattering has been considered early on, and it was shown by Gell-Mann [1], that for a local interaction and massless neutrinos, $\nu + \gamma \to \nu + \gamma$ is forbidden by angular momentum conservation [2]. This restriction does not apply for virtual photons, massive neutrinos or to higher order in $(m_\ell/m_W)^2$. Dicus and collaborators [3-6] have given cross sections for both massless and massive neutrino photon scattering. Neutrino scattering in a magnetic field has been considered in detail by Mikheev and collaborators [7-10], as well as by other authors [11,12].

For high energy neutrinos, the radiated photons are energetic and the distribution is peaked in the forward direction. The kinematics can be visualized in terms of the 3-momentum $\vec{q}$ of the virtual photons. We take $\vec{q}$ to be directed opposite to the neutrino momentum and $q^0 = 0$. With $E, \vec{p}$ and $\vec{E}', \vec{p}\,'$ the incoming and outgoing neutrino 4-vectors, m_ν the neutrino mass and ω' the scattered photon energy,

$$\omega' = \frac{E}{1 + (m_\nu/2\gamma|q|)(1+\gamma^2\theta^2)} \tag{1}$$

Here $\gamma = E_\nu/m_\nu$ and θ is the scattering angle measured from the forward direction.

In the following section we present the scattering rate as calculated in the standard model. This is followed by numerical estimates of expected rates for operating and planned accelerators. We conclude with a discussion of effects that would manifest themselves should neutrinos possess a magnetic moment.

2. Scattering Rate in the Standard Model

The relevant amplitudes are shown in Fig.1. For small momentum transfers, $s/m_W^2 \ll 1$, the boson propagators can be contracted to the low-energy 4-fermion interaction. It is also evident from the graphs, that the dominant contribution will come from the lowest mass fermion, in the loop the electron. The W-exchange graph allows for mixing of the neutrino flavor eigenstates, specified by a unitary matrix $U_{\ell\alpha}$, where ℓ, α are the flavor, mass, eigenstate indices. The external field appears in the neutrino rest-frame as time independent crossed electric and magnetic fields. The exact solution of the wave equation in these fields is used for the lepton propagator [7] instead of the virtual photon coupling to the external field.

The scattering rate (probability per unit time) is expressed in terms of the invariant χ_e defined through

$$\chi_e^2 = \frac{e^2(p_\nu F F p_\nu)}{m_e^6} = \left(\frac{E_\nu}{m_e}\right)^2 \left(\frac{B}{B_e}\right)^2 = E_\nu^2 \left(\frac{eB}{m_e^3}\right)^2 \tag{2}$$

where $B_e = 4.41 \times 10^9$ T is the Schwinger critical field [13], and m_e the electron mass. Three different cases can be identified. First, for massless neutrinos, $m_\nu = 0$, we find

$$\Gamma = \frac{\alpha}{16\pi} \frac{(G_F m_e^2)^2}{(2\pi)^3} \left[\frac{5}{216}\right] \frac{m_e^2 \chi_e^6}{E_\nu} \tag{3}$$

In the second case the neutrino is assumed massive, $m_\nu \neq 0$. In the absence of mixing the scattering rate is given by

$$\Gamma = \frac{\alpha}{16\pi} \frac{(G_F m_e^2)^2}{(2\pi)^3} \left[\frac{185 \cdot 11}{9 \cdot (30)^3}\right] \frac{m_\nu^2 \chi_e^4}{E_\nu} \tag{4}$$

In the presence of mixing but if the incident mass eignestate, α, is not changed by the scattering Eq.(4) remains valid but is modified by a factor (given here for 3-flavor mixing)

$$\left|U^*_{e\alpha}U_{e\alpha} - \left(U^*_{\mu\alpha}U_{\mu\alpha} + U^*_{\tau\alpha}U_{\tau\alpha}\right)\right|^2 \tag{4a}$$

This factor arises from the interference of the W and Z_0 contributions.

The third cases arises in the presence of mixing but when a massive neutrino α, decays radiatively to a lighter neutrino β. Assuming that $m_{\nu\alpha} \gg m_{\nu\beta}$, the rate for the process $\nu_\alpha \to \nu_\beta + \gamma$ is now

$$\Gamma = \frac{\alpha}{16\pi}\frac{(G_F m_e^2)^2}{(2\pi)^3}\left[\frac{1}{108}\right]\left(\frac{m_{\nu\alpha}}{m_e}\right)^2 \frac{m^2_{\nu\alpha}\chi_e^2}{E_\nu}\left| U^*_{e\alpha}U_{e\beta}\right|^2 \tag{5}$$

Of course in an experiment the incident as well as the detected neutrinos are usually flavor eigenstate. Thus to use Eq.5 or Eq.4a one has to first project the flavor eigenstate onto it mass eignestates.

The above results remain valid as long as $\chi_e < 1$ which is always the case even at the highest neutrino (beam) energies and for magnetic fields that can be achieved in the laboratory. Results pertaining to larger values of χ_e such as can be reached in the very strong fields of astrophysical environments or for extremely high energy cosmic ray neutrinos are discussed in [9,14].

We also show the neutrino photon-scattering cross section for two special cases: (a) Massless neutrino but the initial photon is virtual. In this case we set (in the lab frame) $q^\mu = \{0, 0, 0, -q\},\ p^\mu = \{E_\nu, 0, 0, p_\nu\}$ and find

$$\sigma = \frac{\alpha^2}{16\pi}\ \frac{G_F^2 m_e^2}{(4\pi)^2}\ \left[\frac{5}{54}\right]\ \frac{s}{4m_e^2}\ \left(\frac{q^2}{m_e^2}\right)^2 \tag{6}$$

Here $s = 2E_\nu|q|$ is the square of the cm energy and $q^2 = -q_\mu q^\mu$.

(b) When the incoming photon is real but the neutrino is massive. The cross section for this process has been discussed in detail in ref. [6]. When $s \gg 4m_e^2$ the cross section tends to a constant value

$$\sigma \simeq \frac{\alpha^2}{16\pi}\ \left(\frac{G_F^2 m_\nu^2}{4\pi^2}\right)\ \left(\frac{s - m_\nu^2}{s}\right)^2 \tag{7}$$

Eqs.6,7 can be used in conjunction with the virtual photon formalism alluded to previously to obtain estimates of the scattering rates in a magnetic field.

3. Numerical Estimates

The range of variables entering eqs.(3-5) is restricted by the experimental possibilities. We will therefore consider only the following values (in Gaussian units)

$$
\begin{array}{lcll}
B & = & 2.2 \text{ T } \simeq 2.2 \times 693 & \text{eV}^2 \\
L & = & 10 \text{ m} \simeq 5 \times 10^7 & \text{eV}^{-1} \\
E_\nu & = & 50 \text{ GeV} &
\end{array}
$$

It follows that $\chi_e \simeq 0.5 \times 10^{-4}$ and for the purposes of this estimate we have set $U^*_{e\alpha} U_{e\beta} = 1$ in Eq.5. The resulting probabilities for radiative scattering per incident neutrino are shown in Fig.2, as a function of neutrino mass. It is important to note that Eqs.(3-5) have been obtained by using the amplitude corresponding to only one of the three possible cases. Thus for a given neutrino mass the probability for radiative scattering is given by the dominant contribution: see for instance Fig.2.

We note that for $m_\nu \gtrsim 100$ eV the dominant process is the radiative decay catalyzed by the presence of the magnetic field. The probabilities shown in Fig.2 are to be compared to the available integrated fluxes of high energy neutrinos. The MINOS beam at Fermilab will soon deliver 10^{18}, 50 GeV neutrinos per year, mainly ν_μ. A future 50 GeV neutrino factory could deliver, under optimal conditions 10^{21} neutrinos in one year at $E_\nu \simeq 20$ GeV. In either case these fluxes are too low to lead to observable effects unless $m_\nu > 100$ MeV.

4. Magnetic Moment Interactions

A Majorana neutrino or a massless Dirac neutrino can have no magnetic moment. A massive Dirac neutrino has a magnetic moment which arises in the standard model from loop corrections. To leading order in $(m_\ell/m_W)^2$ it has the value [15]

$$
\mu_\nu = \frac{3eG_F\, m_\nu}{8\pi^2\sqrt{2}} = 3.2 \times 10^{-19} \left(\frac{m_\nu}{1 \text{ eV}}\right) \mu_0 \tag{8}
$$

where μ_0 is the Bohr magneton, $\mu_0 = 5.79 \times 10^{-11}$ MeV/T.

The accepted limits on possible magnetic moments as given by the PDG [16] are

$$
\begin{array}{ll}
\mu_{\nu 1} \gtrsim 10^{-10}\ \mu_0 & \text{electron neutrino} \\
\mu_{\nu 2} \gtrsim 10^{-9}\ \mu_0 & \text{muon neutrino} \\
\mu_{\nu 3} \gtrsim 10^{-6}\ \mu_0 & \tau\text{−neutrino}
\end{array}
$$

These limits are obtained most directly from the shape and rate of the spectrum in νe and $\bar{\nu} e$ scattering (see for instance Ahrens et al [17]). There are also astrophysical limits based on the cooling rate of stars and on the observation of the neutrino burst from SN1987A. Such limits are in the range of $(10^{-10}$ to $10^{-12})\mu_0$ for all three neutrino mass eigenstates. We discuss three possible manifestations of a magnetic moment interaction in a magnetic field. For laboratory field strengths these processes are far from reaching the value predicted by Eq.8, nor can they improve on the existing limits listed previously.

The simplest interaction is the precession of the spin vector which modifies the helicity state of the neutrino and thus alters its weak interaction rate. Spin rotation is due to the different time evolution of the two spin states projected onto the magnetic field; the neutrino momentum vector is assumed to be perpendicular to the magnetic field. The rotation angle is

$$\theta = 2\mu_\nu BL \tag{9}$$

where L is the length of the field. The fractional change in the weak cross-section is then

$$\Delta\sigma/\sigma \simeq 1 - \cos^2\theta$$

and for small rotation angles

$$\Delta\sigma/\sigma \simeq 4\mu_\nu^2 B^2 L^2 \tag{10}$$

For $B = 2$ T and $L = 10$ m a change $\Delta\sigma/\sigma \gtrsim 1\%$ would correspond to a limit $\mu_\nu/\mu_0 \gtrsim 10^{-5}$.

If we allow for mixing, different mass eigenstates would have different magnetic moments. In the presence of flavor mixing among magnetic moment eigenstates, the presence of an axial magnetic field would lift the degeneracy between these eigenstates, and potentially result in field-induced neutrino flavor oscillations. Analyzing this situation in the familiear two neutrino case where we assume the mass and magnetic moment eigenstates to be identical, as in the standard model, we find the flavor oscillation probability:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2(\Delta m_\nu^2 L/4E + \Delta\mu_\nu BL/2)$$

where θ is the mixing angle, $\Delta m_\nu^2 = \left| m_\alpha^2 - m_\beta^2 \right|$ and $\Delta\mu = |\mu_\alpha - \mu_\beta|$. The second term in the time evolution factor is typically smaller than the first term, except in cases wehre the magnetic moment is anomalous, the field strength is extreme or the neutrino is very energetic ($E_\nu = 10^{12}$ GeV when $(m_\alpha + m_\beta) \sim 0.1$ eV and $B = 2$ T).

In a laboratory experiment, one can search for an anomalous neutrino magnetic moment through this process. For $B = 2$ T, $L = 10$ m and if one can detect oscillations at the 10^{-4} level, then for maximal mixing the limit is $\Delta\mu/\mu_0 < 2 \times 10^{-6}$.

The presence of a magnetic moment would also lead to the emission of a high energy photon by magnetic "Compton" scattering as shown in Fig.3. For the cross-section for this process involving real or virtual photons and assuming that $s \gg q^2$, we find

$$\sigma = \frac{\pi\alpha^2}{8m_e^2}\left(\frac{\mu_\nu}{\mu_0}\right)^4 \frac{s}{m_e^2} \tag{11}$$

where s is the cm energy. For a head-on collision with a real photon of momentum q, $s = 4E_\nu q$. Using the virtual photon formalism [14] we can obtain the radiation rate in a magnetic field B

$$\Gamma \simeq \frac{1}{8}\alpha \left(\frac{\mu_\nu}{\mu_0}\right)^4 \frac{m_e^2}{E_\nu} \chi_e^2 \tag{12}$$

where χ_e is the invariant of Eq.3. For $E_\nu = 50$ GeV, and $\chi_e = 0.5 \times 10^{-4}(B = 2.2$ T), $L = 10$ m and $(\mu_\nu \mu_0) = 7 \times 10^{-5}$ the scattering probability is $P \simeq 10^{-20}$. Given the available neutrino fluxes this can be considered as the lowest limit of detection.

Finally we note that the presence of a transition magnetic moment leads to radiative decay. The invariant probability (inverse lifetime in the neutrino rest frame) is [18]

$$\Gamma(\nu_\alpha \to \nu_\beta) = \left(\frac{\mu_{\alpha\beta}}{\mu_0}\right)^2 \mu_0^2 \frac{(\Delta m_{\alpha\beta}^2)^3}{8\pi m_e^3} \tag{13}$$

For instance for $L = 10$ m, $E_\nu = 50$ GeV and $\Delta m_{\alpha\beta}^2 = m_\alpha^2 = 1$ eV^2, the decay probability is

$$P = 3.5 \times 10^{-18} (\mu_{\alpha\beta}/\mu_0)^2 \tag{14}$$

It has been proposed to use an external radio frequency field to induce the transition [19]. As an example, using one of the LEP rf cavities at nominal power levels the probability for a (non-radative) transition is of order

$$P \sim 10^3 (\mu_{\alpha\beta}/\mu_0)^2 \tag{15}$$

Experimentally, the transition leads to a change in the rate of detection of a particular neutrino flavor. This limits the observable probability to $P \gtrsim 10^{-3}$, or
$(\mu_{\alpha\beta}/\mu_0) < 10^{-3}$.

This work was supported in part under DOE grant DE-FG02-91ER40685; The work of N.V.Mikheev was supported in part by the Russian Foundation for Basic Research under the Grant No. 01-02-17334.

References

1. M. Gell-Mann, Phys. Rev. Lett. 6, 70 (1960).
2. C.N. Yang, Phys. Rev. 77, 242 (1950), L.D. Landau, Sov. Phys. Dokl. 60, 207 (1948).
3. D.A. Dicus and W.W. Repko, Phys. Rev. Lett. 79, 569 (1997).
4. D.A. Dicus, C. Kao and W.W. Repko, Phys. Rev. D59, 013005 (1998).
5. D.A. Dicus and W.W. Repko, hep-ph/0003305.
6. D.A. Dicus, W.W. Repko and R. Vega, hep-ph/0006264.
7. LA. Vasilevskaya, A.A. Gvozdev, N.V. Mikheev, Yadern. Fizika 57, 124 (1994), Physics of Atomic Nuclei 57, 117 (1994).
8. A.V. Kuznetsov and N.V. Mikheev, Physics Letters B299, 367 (1993).
9. A.A. Gvozdev, N.V. Mikheev and L.A. Vassilevskeya, Phys. Rev. D54, 5674 (1996).
10. A.V. Kuznetsov and N.V. Mikheev, Physics Letters B394, 123 (1997), A.A. Gvozdev, N.V. Mikheev and L.A. Vassilevskeya, Physics Letters B410, 211 (1997).
11. A.N. Ioannisian and G.G. Raffelt, Phys. Rev. D55, 7038 (1997), G.G. Raffelt, Physics Reports 320, 319 (1999).
12. R. Shaisultanov, Phys. Rev. Lett. 80, 1586 (1998), H. Giess and R. Shaisultanov, Phys.Rev. D62, 073003 (2000).
13. J. Schwinger, Phys. Rev. 82, 664 (1951); 93, 615 (1954).
14. K.S. McFarland and A.C. Melissinos, "Quantum Electrodynamics and Physics of the Vacuum", G. Cantatore ed. AIP Conference Proceedings 564, 158 (2001).
15. K. Fujikawa and R.E. Shrock, Phys. Rev. Lett. 45, 963 (1980).
16. D.E. Groom et al. (Particle Data Group), Eur. Phys. Jour. C15, 1 (2000).
17. L.A. Ahrens et al., Phys. Rev. D41, 3297 (1990).
18. M.C. Gonzales-Garcia, F. Vannucci and J. Castromonte, Phys. Lett. B373, 153 (1996).
19. S. Matsuki and K.Yamamoto, Phys. Lett. B289, 194 (1992); F. Vannucci, hep-exp/9911025..

Figure Captions

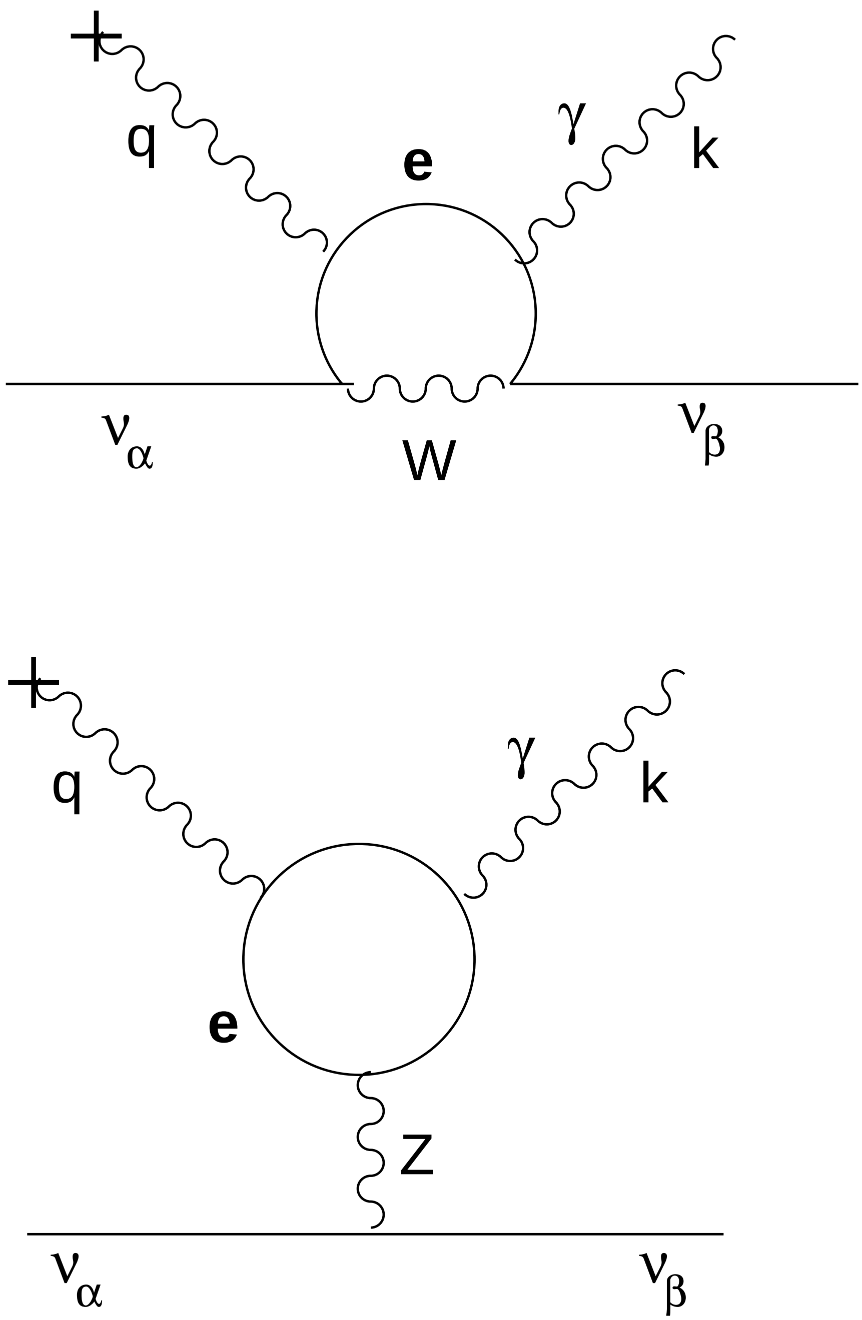

Fig.1 The two lowest order diagrams that contribute to radiative scattering in a magnetic field.

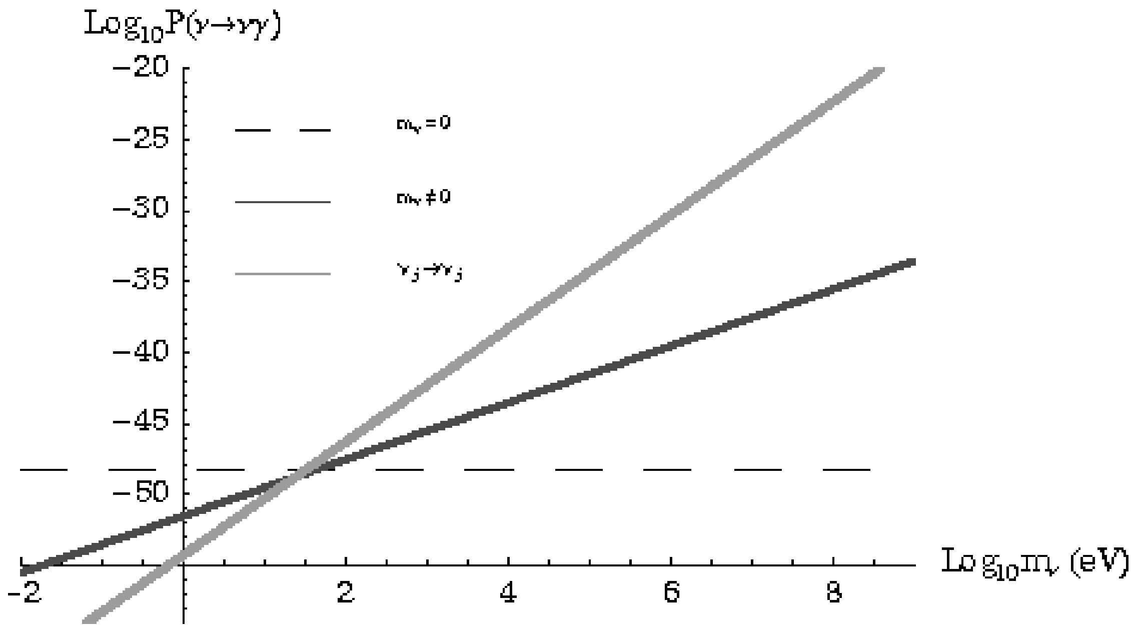

Fig.2 Probability for radiative scattering in a magnetic field $B = 2.2$ T and $L = 10$ m calculated for $E_\nu = 50$ GeV as a function of neutrino mass for the three cases discussed in the text [Eqs.3-5]. The present limits on neutrino masses are [16]: $m_{\nu e} < 7$ eV, $m_{\nu\mu} < 0.17$ MeV, $m_{\nu\tau} < 24$ MeV.

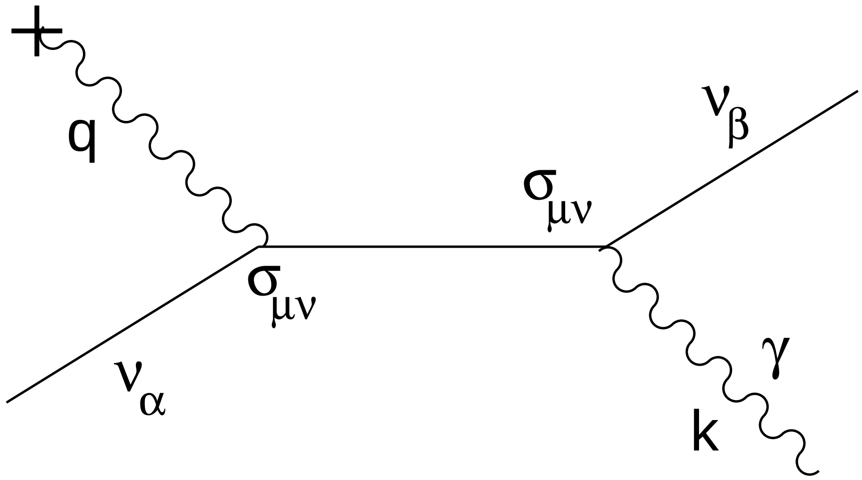

Fig.3 Feynman diagram for radiative scattering in a magnetic field in the presence of a magnetic moment.